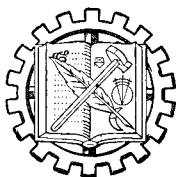


# Dynamical Analogies

*By*

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## PREFACE

Analogies are useful for analysis in unexplored fields. By means of analogies an unfamiliar system may be compared with one that is better known. The relations and actions are more easily visualized, the mathematics more readily applied and the analytical solutions more readily obtained in the familiar system.

Although not generally so considered the electrical circuit is the most common and widely exploited vibrating system. By means of analogies the knowledge in electrical circuits may be applied to the solution of problems in mechanical and acoustical systems. In this procedure the mechanical or acoustical vibrating system is converted into the analogous electrical circuit. The problem is then reduced to the simple solution of an electrical circuit. This method has been used by acoustical engineers for the past twenty years in the development of all types of electro-acoustic transducers. Mechanical engineers have begun to use the same procedure for analyzing the action of mechanisms.

The importance and value of dynamical analogies to any one concerned with vibrating systems have led to a demand for expositions on this branch of dynamics. Accordingly this book has been written with the object of presenting the principles of dynamical analogies to the engineer.

This book deals with the analogies between electrical, mechanical rectilinear, mechanical rotational and acoustical systems. The subject matter is developed in stages from the simple element through to complex arrangements of multielement systems. As an aid in the establishment of these analogies a complete theme is depicted in each illustration.

The text assumes on the part of the reader a familiarity with the elements of alternating circuit theory and physics.

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HARRY F. OLSON

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