The James-Baxandall Passive Tone-Control Network

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Independent adjustment of bass and treble frequencies in high fidelity audio amplifiers is usually accomplished utilizing specially designed tone-control networks. There are versions for these tone controls based only on passive components, such as the ground-referenced James network shown in fig. 1. Among those versions using active devices we must make mention of P.J. Baxandall´s proposal, in which the tone control was devised as a feedback amplifier (refs. 1 and 2).

In this article we shall analyze the James network (also known as the passive Baxandall tone control), obtaining its design equations.

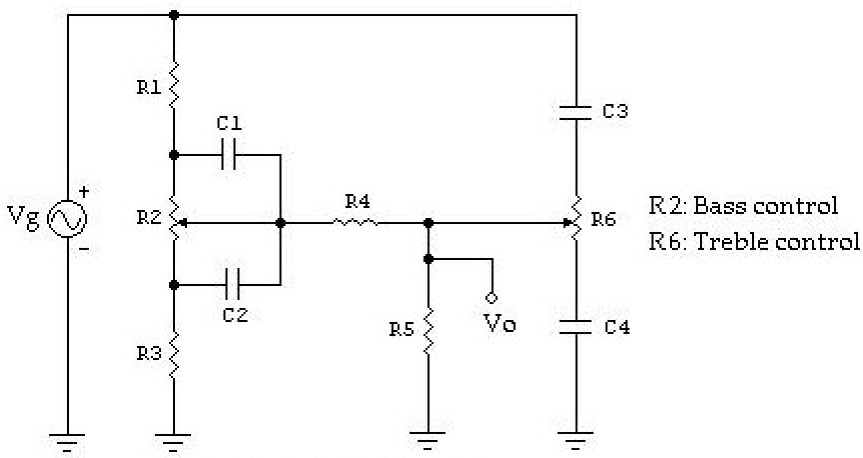

Fig.1 The James tone control

Let´s begin studying the bass control section (fig.2), which has influence over the frequencies below the designed center frequency of the overall James network.

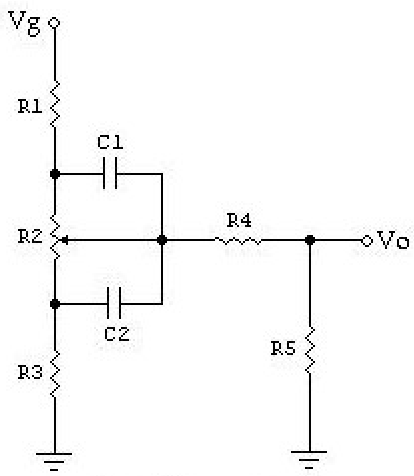

Fig.2 Bass control

Here, R_4 offers some isolation between this stage and the treble control section (frequencies above the center frequency are affected by this control). R_5 represents the input resistance of the amplifier connected to the output of the James network and should be selected such that it imposes no appreciable load on the network. Here we assume that C_3 and C_4 are open circuits at the bass frequencies.

With full bass boost (R_2´s slider at the upper end), the equivalent circuit for the tone control is as depicted by fig.3.

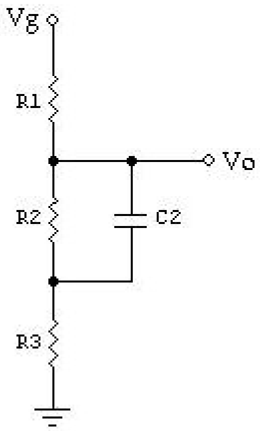

Fig.3 Bass control at full boost

From this figure and knowing that “s” is Laplace´s variable, we obtain:

$$V_0 = V_g \frac{R_3 + R_2 // X_{C2}}{R_1 + R_3 + R_2 // X_{C2}}$$

where:

$$R_2 // X_{C2} = \frac{R_2 \times \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}}$$

$$= \frac{R_2}{sR_2C_2 + 1}$$

Then:

$$V_0 = V_g \frac{R_3 + \frac{R_2}{sR_2C_2 + 1}}{R_1 + R_3 + \frac{R_2}{sR_2C_2 + 1}}$$

After some algebraic work we arrive to:

$$\frac{V_0}{V_g} = \frac{R_3}{R_1 + R_3} \cdot \frac{sR_2C_2 + 1 + \frac{R_2}{R_3}}{sR_2C_2 + 1 + \frac{R_2}{R_1 + R_3}} \qquad \text{...(1)}$$

The gain for high bass frequencies is:

$$A_1 = \frac{R_3}{R_1 + R_3}$$

The gain at low frequencies is:

$$A_2 = \frac{R_2 + R_3}{R_1 + R_2 + R_3}$$

Expression (1) has a zero given by:

$$s_{01} = -\frac{1 + \frac{R_2}{R_3}}{R_2C_2}$$

and a pole:

$$s_{p2} = -\frac{1 + \frac{R_2}{R_1 + R_3}}{R_2C_2}$$

With the tone control adjusted for maximum bass cut (R_2´s slider at the lower end), the equivalent circuit changes to that of fig.4. Again, we neglect the loading effect of R_4 and R_5.

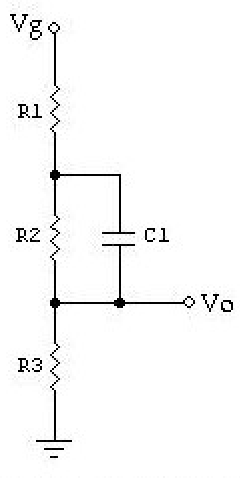

Fig.4 Bass control at full cut

Now the following holds:

$$V_0 = V_g \frac{R_3}{R_1 + R_3 + R_2 // X_{C1}}$$

$$= V_g \frac{R_3}{R_1 + R_3 + \dfrac{R_2}{sR_2C_1 + 1}}$$

A simple algebraic manipulation takes us to:

$$\frac{V_0}{V_g} = \frac{R_3}{R_1 + R_3} \cdot \frac{sR_2C_1 + 1}{sR_2C_1 + 1 + \dfrac{R_2}{R_1 + R_3}} \qquad ...(2)$$

The gain at high bass frequencies is now given by:

$$A_3 = \frac{R_3}{R_1 + R_3}$$

For low frequencies the gain is:

$$A_4 = \frac{R_3}{R_1 + R_2 + R_3}$$

Expression (2) has a zero given by:

$$s_{03} = -\frac{1}{R_2C_1}$$

and a pole:

$$s_{p4} = -\frac{1 + \dfrac{R_2}{R_1 + R_3}}{R_2C_1}$$

The gain ratio at low frequencies is:

$$\frac{A_2}{A_4} = 1 + \frac{R_2}{R_3}$$

For a 40dB control range the following relationship must be satisfied:

$$1+\frac{R_2}{R_3}=100 \qquad \text{...(3)}$$

Then:

$$R_2=99R_3\approx 100R_3$$

On the other hand, at maximum bass boost, the ratio of the gains for low and high bass frequencies is:

$$\frac{A_2}{A_1}=\frac{\dfrac{R_2+R_3}{R_1+R_2+R_3}}{\dfrac{R_3}{R_1+R_3}}$$

$$=\frac{R_1+R_3}{R_3}\cdot\frac{R_2+R_3}{R_1+R_2+R_3}$$

According to expression (3):

$$R_2+R_3=100R_3$$

Then:

$$\frac{A_2}{A_1}=\frac{R_1+R_3}{R_3}\cdot\frac{100R_3}{R_1+100R_3}$$

$$=\frac{100(R_1+R_3)}{R_1+100R_3}$$

For a 20dB bass boost:

$$\frac{100(R_1+R_3)}{R_1+100R_3}=10$$

Solving for R_1 we arrive to the following relationship:

$$R_1=10R_3$$

We may check that:

$$\frac{A_3}{A_4}=\frac{R_3}{R_1+R_3}\cdot\frac{R_1+R_2+R_3}{R_3}$$

$$=10$$

this is, the ratio of the high-frequency and low-frequency bass gains is also 20dB at maximum bass cut.

For best symmetry in the response curves we must choose $s_{01} = s_{p4}$. Therefore:

$$\frac{1+\dfrac{R_2}{R_3}}{C_2} = \frac{1+\dfrac{R_2}{R_1+R_3}}{C_1}$$

Substituting the already obtained relationships between the resistances yields:

$$\frac{100}{C_2} = \frac{1+\dfrac{R_2}{11R_3}}{C_1}$$

$$= \frac{1+\dfrac{99}{11}}{C_1}$$

$$= \frac{10}{C_1}$$

Then, $C_2 = 10C_1$.

The corresponding Bode plot of the bass response at full boost and full cut is shown below.

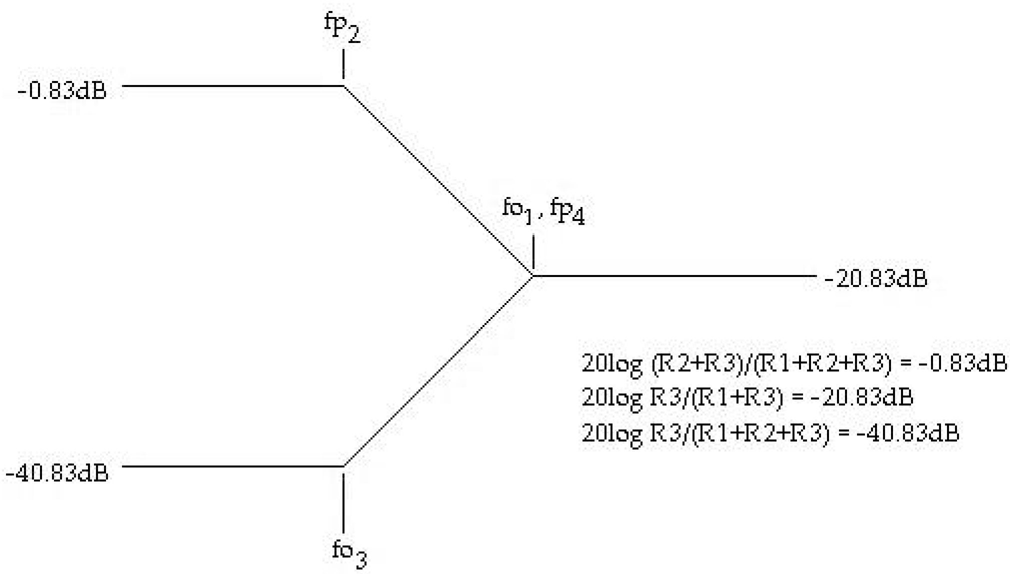

Fig.5 Bode plot of the bass response

Next, we will analyze the treble control. We have already stated that the treble frequencies are those above the center frequency for which the James network is designed. We may consider C_1 and C_2 as short circuits at these frequencies. Therefore, R_1 and R_3 along with R_4 are part of the treble network (fig.6).

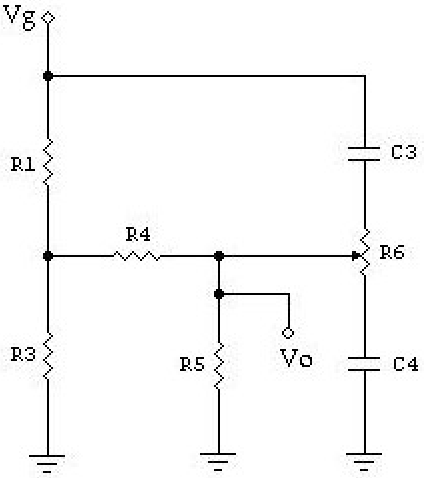

Fig.6 Treble control

In order to facilitate calculations, we shall first find the Thevenin equivalent for V_g, R_1 and R_3. In fig.7, the equivalent Thevenin voltage source, V_1, is given by:

$$V_1 = V_g \frac{R_3}{R_1 + R_3}$$

$$= \frac{V_g}{11} \qquad \text{...(4)}$$

The equivalent Thevenin resistance, R_{TH}, is:

$$R_{TH} = R_1 // R_3$$

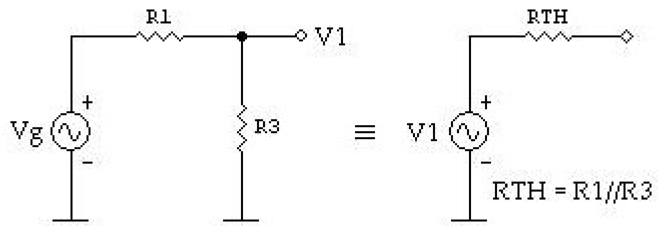

Fig.7 Thevenin equivalent for Vg, R1 and R3

With the treble control adjusted for maximum boost (R_6´s slider at the upper end), and assuming the current through resistor R_6 is much smaller than that flowing through R_4, we may state the following:

$$\frac{V_1 - V_0}{R_{TH} + R_4} = (V_0 - V_g)sC_3$$

if $R_5 >> R_4$.

Substituting the value given by expression (4) for V_1:

$$\frac{\frac{V_g}{11} - V_0}{R_{TH} + R_4} = (V_0 - V_g)sC_3$$

Rearranging this equation we get:

$$V_g\left[\frac{1}{11} + sC_3(R_{TH} + R_4)\right] = V_0\left[1 + sC_3(R_{TH} + R_4)\right]$$

Then:

$$\frac{V_0}{V_g} = \frac{\frac{1}{11} + sC_3(R_{TH} + R_4)}{1 + sC_3(R_{TH} + R_4)} \qquad ...(5)$$

At sufficiently low treble-frequencies:

$$\frac{V_0}{V_g} = \frac{1}{11}$$

the same as -20.83dB.

At sufficiently high treble-frequencies:

$$\frac{V_0}{V_g} = 1$$

or 0dB.

Expression (5) has a zero given by:

$$s_{05} = -\frac{1}{11 \cdot (R_{TH} + R_4)C_3}$$

and a pole:

$$s_{p6} = -\frac{1}{(R_{TH} + R_4)C_3}$$

With the treble control adjusted for maximum cut (R_6´s slider at the lower end) we may write:

$$\frac{V_1 - V_0}{R_{TH} + R_4} = sC_4 V_0$$

Then:

$$\frac{V_0}{V_1} = \frac{1}{1 + s(R_{TH} + R_4)C_4}$$

From the above equation and bearing in mind (4):

$$\frac{V_0}{V_g} = \frac{1}{11 \cdot [1 + s(R_{TH} + R_4)C_4]} \qquad ...(6)$$

At sufficiently low treble-frequencies:

$$\frac{V_0}{V_g} = \frac{1}{11}$$

or -20.83dB.

At sufficiently high treble-frequencies:

$$\frac{V_0}{V_g} \to 0$$

Expression (6) has a pole given by:

$$s_{p7} = -\frac{1}{(R_{TH} + R_4)C_4}$$

For best symmetry in the response curves we must choose $s_{p7} = s_{05}$. Therefore:

$$\frac{1}{(R_{TH} + R_4)C_4} = \frac{1}{11 \cdot (R_{TH} + R_4)C_3}$$

yielding $C_4 = 11C_3$.

Fig.8 shows the corresponding Bode plot of the treble response at full boost and full cut.

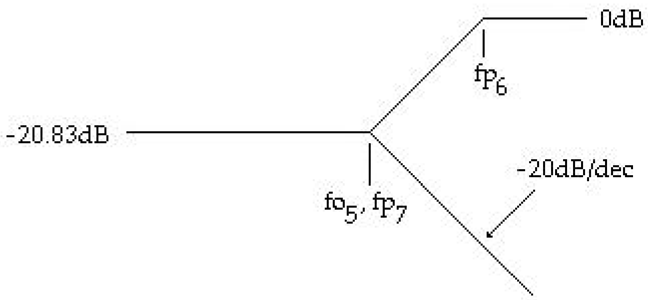

Fig.8 Bode plot of the treble response

A convenient value for R_6 must now be computed. We need some knowledge on the constraints the network imposes before we may neglect the current through R_6, as compared to that circulating through R_4 ($I_{R6} \ll I_{R4}$). I_{R4} is given by the following expression when the treble control is adjusted for full cut:

$$I_{R4} = \frac{\frac{V_g}{11} - V_0}{R_{TH} + R_4}$$

and I_{R6} by:

$$I_{R6} = \frac{V_g - V_0}{R_6 + \frac{1}{sC_3}}$$

Therefore, the requirement that must be satisfied is:

$$\frac{V_g - V_0}{\left| R_6 + \frac{1}{sC_3} \right|} \langle\langle \frac{\frac{V_g}{11} - V_0}{R_{TH} + R_4}$$

The worst case for the above inequality occurs when s approaches infinity. Translated to our analysis, at sufficiently high treble-frequencies. Then, it is required that:

$$\frac{\frac{V_g}{11} - V_0}{V_g - V_0} \rangle\rangle \frac{R_{TH} + R_4}{R_6}$$

or equivalently:

$$\frac{\frac{1}{11} - \frac{V_0}{V_g}}{1 - \frac{V_0}{V_g}} \rangle\rangle \frac{R_{TH} + R_4}{R_6}$$

At sufficiently high frequencies:

$$\frac{V_0}{V_g} \to 0$$

It is clear then that the following must be satisfied:

$$\frac{1}{11} \rangle\rangle \frac{R_{TH} + R_4}{R_6}$$

Then:

$$R_6 \rangle\rangle 11(R_{TH} + R_4) \qquad ...(7)$$

If we conduct a similar analysis when the treble control is at full boost, we would find that the condition to be met is R_6>>1.1R_4. Hence, expression (7) prevails.

We would like to arrive at easy-to-use design formulae, so, before presenting a design example for the James network we shall try to simplify the expressions for f_{01}, f_{p2}, f_{03}, f_{p4}, f_{05}, f_{p6} and f_{p7}. First, for convenience, the relationships between the components' values will be repeated. These are:

$$R_1 = 10R_3$$

$$R_2 = 99R_3$$

$$C_2 = 10C_1$$

$$C_4 = 11C_3$$

f_{01} is given by:

$$f_{01} = \frac{1}{2\pi} \cdot \frac{1 + \frac{R_2}{R_3}}{R_2 C_2}$$

$$= \frac{1}{2\pi} \cdot \frac{100}{R_2 C_2}$$

$$= \frac{1}{2\pi} \cdot \frac{100}{99 R_3 C_2}$$

$$\approx \frac{1}{2\pi} \cdot \frac{1}{R_3 C_2} \qquad ...(8)$$

f_{p2} is given by:

$$f_{p2} = \frac{1}{2\pi} \cdot \frac{1 + \dfrac{R_2}{R_1 + R_3}}{R_2 C_2}$$

$$= \frac{1}{2\pi} \cdot \frac{10}{R_2 C_2}$$

$$= \frac{1}{2\pi} \cdot \frac{10}{99 R_3 C_2}$$

$$\approx \frac{1}{2\pi} \cdot \frac{1}{10 R_3 C_2} \qquad \text{...(9)}$$

f_{03} is given by:

$$f_{03} = \frac{1}{2\pi} \cdot \frac{1}{R_2 C_1}$$

$$= \frac{1}{2\pi} \cdot \frac{10}{R_2 C_2}$$

$$= f_{p2}$$

$f_{p4} = f_{01}$, required for the desired symmetry on the response curves.

f_{05} is given by:

$$f_{05} = \frac{1}{2\pi} \cdot \frac{1}{11(R_{TH} + R_4)C_3}$$

$$= \frac{1}{2\pi} \cdot \frac{1}{11(R_1 // R_3 + R_4)C_3}$$

$$= \frac{1}{2\pi} \cdot \frac{1}{11\left(\frac{10}{11} R_3 + R_4\right)C_3}$$

$$= \frac{1}{2\pi} \cdot \frac{1}{(10 R_3 + 11 R_4)C_3} \qquad \text{...(10)}$$

f_{p6} is given by:

$$f_{p6} = \frac{1}{2\pi} \cdot \frac{1}{(R_{TH} + R_4)C_3}$$

$$= 11 f_{05}$$

$f_{p7} = f_{05}$, required for the desired symmetry on the response curves.

The center frequency of the James network is taken as the geometric mean of f_{01} and f_{05}, this is:

$$f_c = \sqrt{f_{01} \times f_{05}} \qquad \text{...(11)}$$

which coincides with the frequency of the minimum of the amplitude response curve when the bass and treble controls are at full boost. Agrees also with the frequency of the maximum of the amplitude response curve when both controls are at full cut. Usually, 1kHz is adopted as the center frequency.

A Design Example

Let us suppose we wish to design a tone control for a transistorized piece of equipment. An adequate value for R_1 is 10k ohms. R_3 will then be a 1k ohm-resistor and R_2 a 100k ohm-potentiometer (standard value).

It is convenient that f_{01} and f_{05} be separated one decade in frequency. Then, from expression (11) we may obtain that:

$$f_c = \sqrt{10} f_{01}$$

Being f_c = 1kHz, we find that f_{01} must be 316Hz. The value for f_{05} will then be 3.16kHz.

From expression (8) we obtain for C_2 a value of 503.65nF. Then, C_1 should have a capacitance of 50.36nF. According to (9), f_{p2} = 31.6Hz.

R_5 is taken equal to 5 times R_2 in order to avoid loading effects on the bass network. Therefore, R_5 = 500k ohms. R_4 must be chosen such that an affordable value for R_6 is obtained when using inequality (7). If we make R_4 = 5k ohms, then R_6 must satisfy the condition R_6>>65k ohms. We may adopt a value of 500k ohms for R_6.

From expression (10), with the value chosen for R_4 we obtain C_3 = 774.85pF and C_4 = 8.52nF.

Finally, in order to avoid additional attenuation in the circuit, the output resistance of generator V_g (source resistance) must be made some 20 times smaller than R_1.

Simulation of the frequency response of the James network

Tone Stack Calculator 1.3 is an excellent software program that can be used for simulation of tone-control networks. It may be downloaded from:
http://www.duncanamps.com/tsc/

Two simulations have been made using this program. The first simulation uses the above calculated component values and the second one, standard capacitor values for the circuit. No major variation on frequency response has been observed between them. Simulation results are shown below.

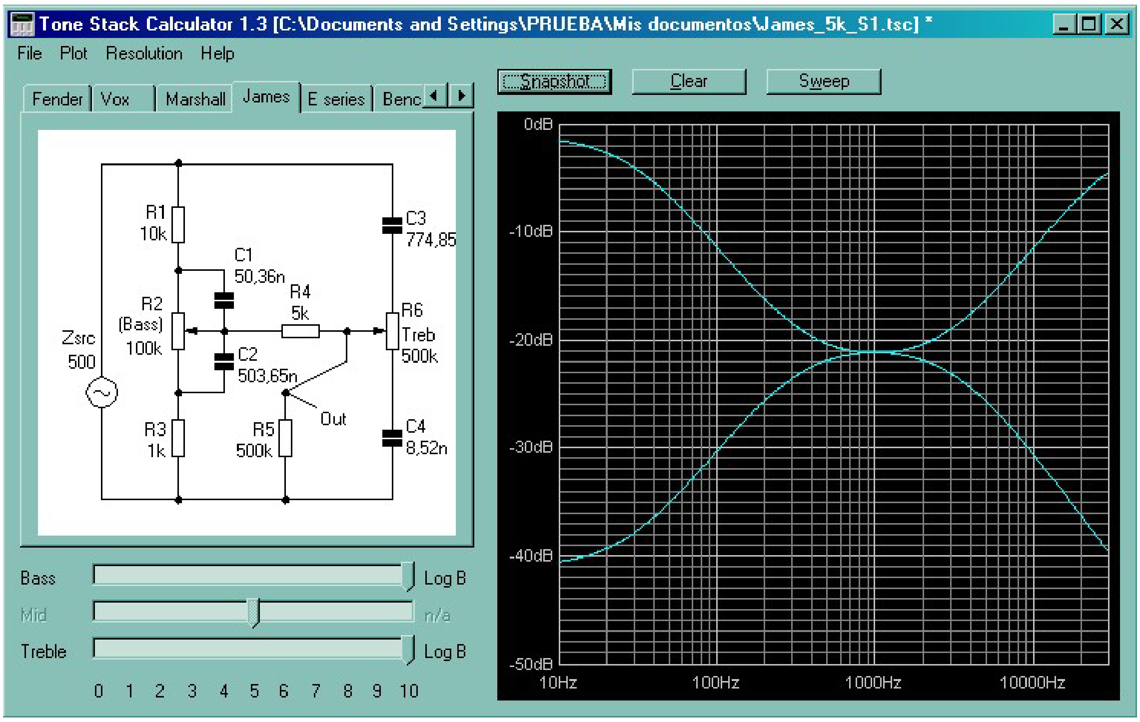

Fig.9 Frequency response of the James network with the bass and treble controls at full boost (upper) and full cut (lower).

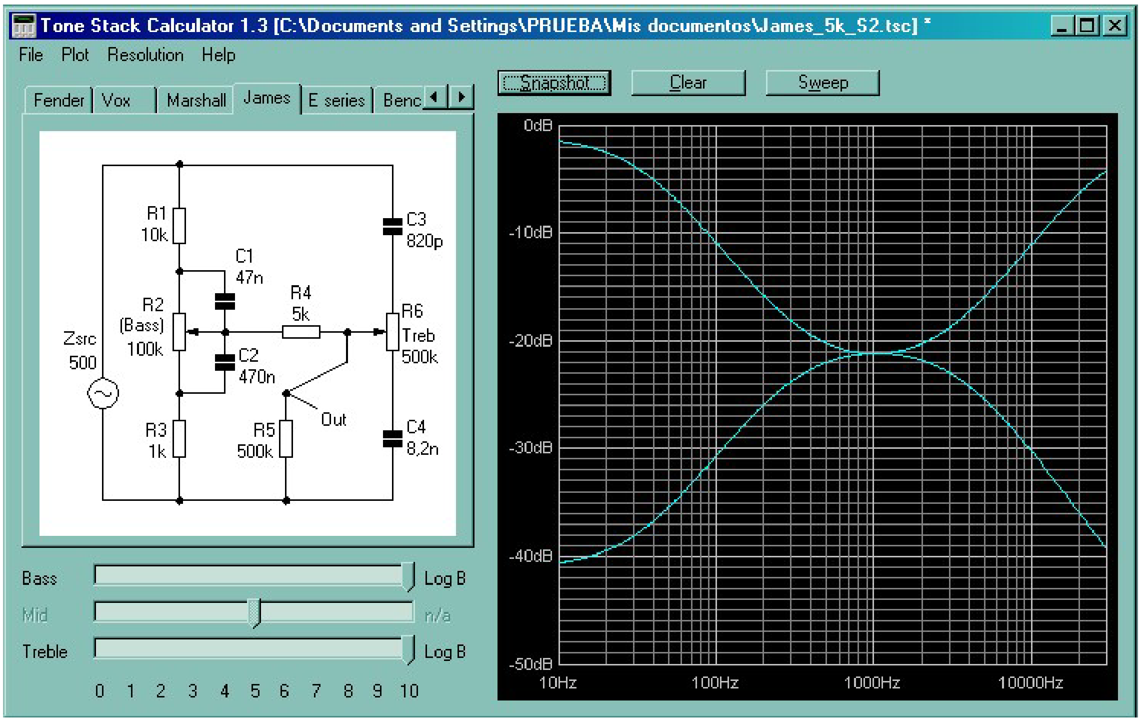

Fig.10 Frequency response of the James network when standard capacitance values for C_1, C_2, C_3 and C_4 are used.

References

1. Baxandall, P.J. “Negative feedback tone control – independent variation of bass and treble without switches” W.W. 58.10 (Oct. 1952) 402. Correction 58.11 (Nov. 1952) 444.

2. Vargas Patrón, Ramón “Red activa de control de tono”
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